Standard Deviation

The Standard Deviation is a measure of **how spread out numbers are**.

You might like to read this simpler page on Standard Deviation first.

But here we explain **the formulas**.

The symbol for Standard Deviation is **σ** (the Greek letter sigma).

This is the formula for Standard Deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Say what? Please explain!

OK. Let us explain it step by step.

Say we have a bunch of numbers like 9, 2, 5, 4, 12, 7, 8, 11.

To calculate the standard deviation of those numbers:

- 1. Work out the Mean (the simple average of the numbers)
- 2. Then for each number: subtract the Mean and square the result
- 3. Then work out the mean of **those** squared differences.
- 4. Take the square root of that and we are done!

The formula actually says all of that, and I will show you how.

The Formula Explained

First, let us have some example values to work on:

Example: Sam has 20 Rose Bushes.

The number of flowers on each bush is

9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4

Work out the Standard Deviation.

Step 1. Work out the mean

In the formula above **μ** (the greek letter "mu") is the mean of all our values ...

Example: 9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4

The mean is:

*9+2+5+4+12+7+8+11+9+3+7+4+12+5+4+10+9+6+9+4***20**

= *140***20** = **7**

So:

μ = 7

Step 2. Then for each number: subtract the Mean and square the result

This is the part of the formula that says:

$$(x_i - \mu)^2$$

So what is x_i ? They are the individual x values 9, 2, 5, 4, 12, 7, etc...

In other words x_1 = 9, x_2 = 2, x_3 = 5, etc.

So it says "for each value, subtract the mean and square the result", like this

Example (continued):

$(9 - 7)^2 = (2)^2 =$ **4**

$(2 - 7)^2 = (-5)^2 =$ **25**

$(5 - 7)^2 = (-2)^2 =$ **4**

$(4 - 7)^2 = (-3)^2 =$ **9**

$(12 - 7)^2 = (5)^2 =$ **25**

$(7 - 7)^2 = (0)^2 =$ **0**

$(8 - 7)^2 = (1)^2 =$ **1**

... etc ...

And we get these results:

4, 25, 4, 9, 25, 0, 1, 16, 4, 16, 0, 9, 25, 4, 9, 9, 4, 1, 4, 9

Step 3. Then work out the mean of those squared differences.

To work out the mean, **add up all the values** then **divide by how many**.

First add up all the values from the previous step.

But how do we say "add them all up" in mathematics? We use "Sigma": Σ

The handy Sigma Notation says to sum up as many terms as we want:
Sigma Notation

We want to add up all the values from 1 to N, where N=20 in our case because there are 20 values:

Example (continued):

$$\sum_{i=1}^{N}(x_i - \mu)^2$$

Which means: Sum all values from $(x_1-7)^2$ to $(x_N-7)^2$

We already calculated $(x_1-7)^2=4$ etc. in the previous step, so just sum them up:

= 4+25+4+9+25+0+1+16+4+16+0+9+25+4+9+9+4+1+4+9 = **178**

But that isn't the mean yet, we need to **divide by how many**, which is done by **multiplying by 1/N** (the same as dividing by N):

Example (continued):

$$\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2$$

Mean of squared differences = (1/20) × 178 = **8.9**

(Note: this value is called the "Variance")

Step 4. Take the square root of that:

Example (concluded):

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}$$

σ = √(8.9) = **2.983...**

DONE!